Numbers, culture and I.T.

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1. History of numbers and information technologies

My work about history of mathematics

1990 Lectures on history of numbers for in-service teachers in the I.R.E.M.

1991-2000 Teaching history of mathematics for pre-service teachers, on various topics (Euclid, Archimed, numbers, beginning of algebra, measure ...)

1996-1999 Teaching history of numbers for students of the first academic year in an *option culturelle*.

1998 Publication of my book *La fabuleuse histoire des nombres* for a large, non specialized public.

1998 Beginning to use I.T. (information technologies), both in university and in a training college and creation of a multimedia laboratory the L.A.M.I.A. in the I.U.F.M. of Lille, in the North of France.

My aims is to present some uses of I.T.

Integration of history How these technologies allow a better integration of history of mathematics in the training of prospective teachers. In my team of research the C.R.E.A.M. *, we create a web site to help young teachers. There are rich pedagogical situations, but also some articles on history of mathematics and a large bibliography with many entries of resources available in French about history of mathematics.

Role of simulations Another team of research Lilimath has created discovery workshops and also a Mathematical exploration park. With an example of use of animations, linked to mathematics used for a seaman, i want to give an idea of the possibilities offered by I.T.

Power of iconography

In july, i have been invited to present my book in holiday centers. How to speak of history of math to a large public ? For a presentation of the writing of numbers in different civilization, i used an abundant iconography that should be useful for teachers with their pupils. I am going to give an idea of the power of this images by showing without translation some of the material used for this presentation.

Creation of web sites about history

I am going to present a wonderful exhibition about history of writing, created by the *Bibliothèque Nationale de France* available on the web.

2. Numbers and culture

Collectively, the relationship enjoyed with numbers is a strange one, consisting partly of fascination, partly of rejection :

- fascination when the precision of numbers, for example, establishes a claim as a *scientific truth* ;
- rejection when the shadow of a numerical value alarms a television presenter concerned about his audience rating.

The ambiguity of the response ought to give for thought, given the great importance numbers of measurement and evaluation have in our societies ; estimations, forecasts, simulations : our lives are, and will always be, influenced by measurements, calculations and comparisons.

Numbers form part of our basic contemporary culture

Are we aware of this when associating, as we tend to, literary and artistic cultures and domains, and opposing them to the sciences ?

The origin of this opposition is to be sought partly in the way science is taught, with truths handed down in isolation from their history and the ways in which they were created.

Mathematics, for example, does not seem to be the work of human hands, but to have fallen out of the sky, perfect and complete, as though it had existed since time immemorial.

Yet it too is the work of successive generations and civilisations ; it too has had its changes, dead ends, progressions and controversies.

Current research in history of maths

Researchers and teachers from all over the world are working. One of their aims is to help change the way mathematics is taught and its relationship to the culture of our societies. For this to come about, that history must be accessible to the majority.

At all levels of education, mathematical activities can be developed that are rich and significant from a scientific standpoint and situate the mathematics of each age in its cultural context.

My personal work

This is one of the aims of my work, which is not based on learned mathematics but on the basic knowledge that everyone possesses : numbers, whole numbers, fractions, rational and irrational numbers, positive and negative integers ...

When and how did the numbers we use appear in history ? In what civilisation ?

Counting and measuring

For numbers, the questions are very easy to formulate. For each civilisation, the forms of knowledge considered essential for teaching can be traced :

– knowing how to read,
– how to write,
– and how to count.

Oddly enough, in the school history syllabus in France, numbers - the ways in which they are represented and used - do not seem to be considered part of the cultural context of their age. Current syllabuses are limited to *reading and writing*.

Numbers go back to the dawn of humanity

Men spoke before writing, and counted before writing down their calculations.

As linguists show, number systems, like kinship systems, are so stable that they can be used to compare languages.

Counting has gone through some difficult stages : going from *one*, *two*, *many* to counting up to ten, counting beyond ordinary numbers or beyond a thousand, for example.

Writing marks the passage from prehistory to history

and mathematics figures in the oldest written documents known to us : the earliest traces of writing in Sumeria are associated with counting tables. On these same clay tables one can follow the birth and evolution of the earliest written signs, the passage from pictogramme to writing proper, and the birth and evolution of a number system.

The power of iconography

This is the material used for my lectures in holiday centers in France.

Wonderful exhibition at the French National Library (Bibliothèque Nationale de France) on *l'aventure des écritures*

3. The History of Numbers in Relation to Teaching

Mathematics and cultures

One can trace the birth of mathematics in the Sumerian, Egyptian, Greek and Arab civilisations, limiting oneself to the civilisations from which we are descended, and then go on to present mathematical methods very different from our own, which did not prevent them solving interesting and difficult problems, as can be seen from Indian, Chinese and Japanese mathematics up to this century.

Our mathematical method

The mathematical method we use was invented in Greece. The public discussions citizens held on the affairs of the city introduced the habit of argumentation. Building on the thoughts of philosophers like Plato and Aristotle, mathematicians introduced into mathematics the deductive method in which, starting out from a few basic propositions admitted as premises, each proposition had to be the object of a demonstration.

This type of method, as all mathematicians know, does not correspond to the way in which the properties in question were discovered, but is an a posteriori reconstruction to prove the accuracy of those propositions. This method has the enormous drawback of concealing the process by which mathematics is created.

4. The evolution of the concept of numbers

What is a number ? I try to answer this question for each of the ancient or medieval civilisations that has directly influenced our own, and then to throw some historical light on each sort of number which currently forms part of our general culture.

Negative integers

They are standard usage today ; you have only to listen to a weather forecast on a very cold day to convince yourself of this.

Nevertheless, they were accepted only after several centuries of controversy among mathematicians. It was the growth of calculus which led to the introduction of negative and imaginary numbers.

It was a tour de force on the part of mathematicians : first they introduce numbers which they themselves qualify as impossible or imaginary, then they use them as pure symbols that, though devoid of meaning, allow them to obtain interesting results.

Integers

They were the first numbers to be used and thought out in arithmetic. What, then, is there for us to leam about them ? Surely, everything has aIready been said ?

By carrying out a guided tour of arithmetics with Gauss to discover certain mathernatical properties of the unlimited decimal developments of rational numbers, one discovers properties that are at the heart of coding problems in data processing.

This bears out one of the most astonishing properties of mathematics : the calculations made by Gauss in the early nineteenth century for altogether different reasons, two centuries later have furnished computer scientists with a powerful tool.

What is an irrational number ?

Is it a number ? Why is it called irrational ? Mathematicians have a habit of taking everyday words and using them to describe mathernatical objects.

The problem of irrationality was a major theme in Greek thought. The answers given by Greek mathematicians in Euclid's Elements have conditioned the growth of mathematics for more than 2,000 years.

On the question of whether or not irrational numbers are numbers, quotations showing the oppositions and controversies of seventeenth and eighteenth century mathematicians give a different picture of mathematics.

Real numbers

It was not until the last century that mathematicians felt the need to introduce a strict order of exposition. The study of real numbers reveals the influence of the growth of analysis on the elaboration of the concept of real numbers, and presents the invention of set theory at the end of the last century in the work of Cantor which marked the reinstation of infinity in mathematics.

Through fairly lengthy quotations clarifying their conception of numbers, one can convince readers that any difficulties in understanding or misgivings they may have had can be explained by seeing those encountered by mathematicians themselves.

Infinity, the discrete and the continuous

In all ages, mathematicians have been confronted with the problem of infinity. Its use in mathematics, with the growth of analysis, is first of all a fact that enriched mathematics and science with a wealth of results.

At the end of this journey through time we are confronted with the same questions that the Greeks had asked about the relationship between the discrete and the continuous.

Likewise, the use of infinity, along with non-standard analysis, which provides logical justification for the infinitely small and the infinitely great (ousted from mathematics in the previous century), is still being studied.

In the meantime, however, mathematics has proved extraordinarily efficient, making possible the growth of physics and computer science by furnishing them with the abstract tools they needed.

Contrary to simplistic claims, it is precisely the most abstract and seemingly gratuitous developments, the ones made, as Dieudonné put it *for the honour of the human mind*, that are susceptible to integration in fields like physics and computer science.

5. The meaning of what we teach

What meaning does our teaching and the training we give to students really have ? Our teaching is always geared towards increasing specialisation and the acquisition of technical skills. Students acquire fragments of knowledge that they are not necessarily able to develop outside the contexts in which they found them, nor piece together to form something resembling a whole.

Mathematics is difficult

By cutting out the difficulties and providing an effective training for pupils and students, one can, of course, instill technical and algorithmic skills. But is this enough to transmit the meaning of the scientific process ?

For teaching to be meaningful once again, we need to construct a coherent, overall view, and dare reply to the question to which there is so often no answer in teaching :

Why are we doing this ?

with something other than :

It'Il be useful for your studies later on.

6. Role of research about History of maths

For teachers this means doing a great deal of research, for there is no single, straightforward answer to such a question, but more like a whole host of viewpoints to be explored.

It is not a matter of giving a historical form to teaching, but of making students aware that mathematics has a history, that it did not fall out of the sky and that it is a series of answers to fundamental questions which societies have raised and handed down from generation to generation and from people to people.

In mathematics, one unifying theme, in particular, seems essential. It concerns the parallel growth of the construction of numbers and the problem of the measurement of magnitudes, neither of which is openly addressed in teaching at present.

7. Mathematics and culture

You only have to hear how people talk about the role of mathematics today to be convinced that mathematicians must make an effort to put across to a broad public the themes and issues on which their science is reared.

The failure of "modern maths"

The reform in question - one of the aims of the instigators of modern mathemàtics - has been a complete flop, for it was founded on the illusion that access to the great structures uncovered after centuries of work was the *royal road* for access to mathematics. The image this gave of mathematics was of a discipline of axiomatic rigour.

Current tendancy

As a result of that failure, mathematicians have done a great deal of work, especially on the history and epistemology of their discipline. Mathematics researchers today stress creativity and the wealth of topics common to fields both inside and outside mathernatics. Current methods of calculation, for example, allow mathematicians the possibilities of experimentation and conjecture which have immediate repercussions on research.

Researchers in mathematics put far more stress today on creativity than on formal rigour ; they see their discipline as a way of developing the imagination of their students and call for a corresponding change in teaching.

The dangers for teaching today

As can be seen from. totalitarian countries and various fundamentalist movements, a purely technical use of science can easily be harnessed to ends for which it was not intended. We must all ask ourselves what it is, in the way that science is taught, that makes this possible. Dogmatism and the fragmentation of knowledge are certainly responsible.

Intense specialisation has led to enormous progress being made in science, but it has huge drawbacks that are clear at the present time. It is the ethical and democratic responsibility of us all to reflect on and address this problem, as any citizen of a democratic society must do.

Towards a new humanism

Mathematics lessons often leave unpleasant memories, especially among those who have had a more literary education. Bridges should be built, in particular among teachers, between bistory, philosophy, French, (or language), the life sciences and physics, to say nothing of computer science. Renaissance humanists were not specialists, they dared to address every field of knowledge of their day.

The separation between mathematics and physics is a very recent one, dating only from the end of the last century. We should endeavour to address the layman in mathematics, for mathematics is part of our contemporary culture, it influences and is influenced by all kinds of intellectual trends. This is the meaning i attach to my personal work in the history of mathematics.